Analysis of Fiber Orientation using the Fokker-Planck Equation*

Colloquium talk by Dogukan T Karahan

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The prediction of fiber orientation is of practical importance in industrial applications such as paper making, additive manufacturing, and construction as the end-product quality is determined by the fiber orientation. In these applications, the most accurate representation of fiber orientation in Eulerian framework is obtained from the fiber orientation probability density (FOPD) function. The FOPD function is defined over the surface of a ball of unit radius at all points in the domain of interest, as illustrated in the figure below. The computational prediction of the FOPD function necessitates solving a special convection-diffusion equation, the Fokker-Planck (FP) equation. Existing solutions of the FP equation in simple shear flow using the finite-volume method highlighted limitations of centered spatial and explicit temporal integration schemes, particularly in providing accurate results across a wide range of conditions. A new finite-volume solver has been developed to solve the FP equation using second-order centered spatial and temporal integration schemes, a combination that was believed to yield inaccurate results. In this talk, it will be demonstrated that, contrary to expectations, the new solver yields accurate and conservative solutions even for highly non-linear problems.

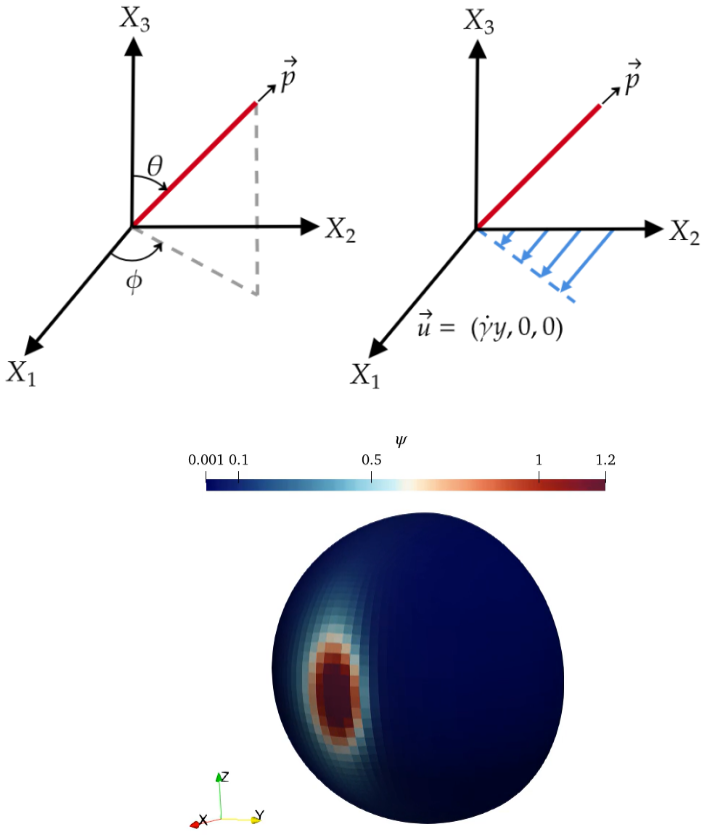

Figure: The orientation of a single fiber and the orientation space (left) and a fiber suspended in simple shear (middle). All possible orientations trace the surface of the unit ball with $\left|\vec{p}\right| \equiv 1$, and

$\dot{\gamma}$ is the strain rate. The coordinates of the orientation space (X_1, X_2, X_3) are coincident with the Cartesian coordinates (x, y, z). The FOPD function, ψ, is defined over the surface of the unit ball for all points in Cartesian space. An example of the FOPD function distribution in simple shear flow (right).

*This work is based on the following publication: Karahan, D. T., Ranjan, D., and Aidun, C. K. (2024). A finite-volume framework to solve the Fokker-Planck equation for fiber orientation kinetics (under review). Journal of Non-Newtonian Fluid Mechanics.