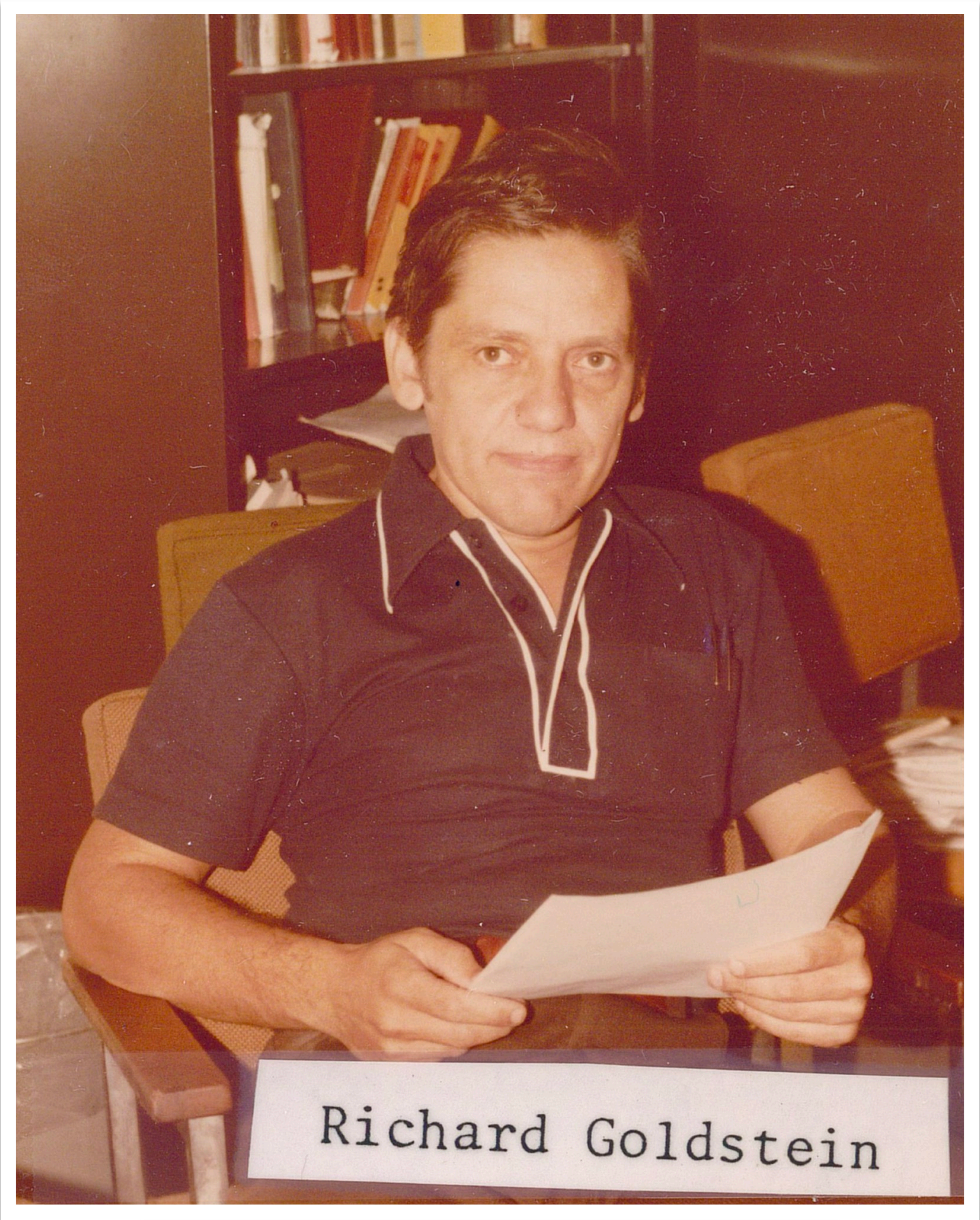
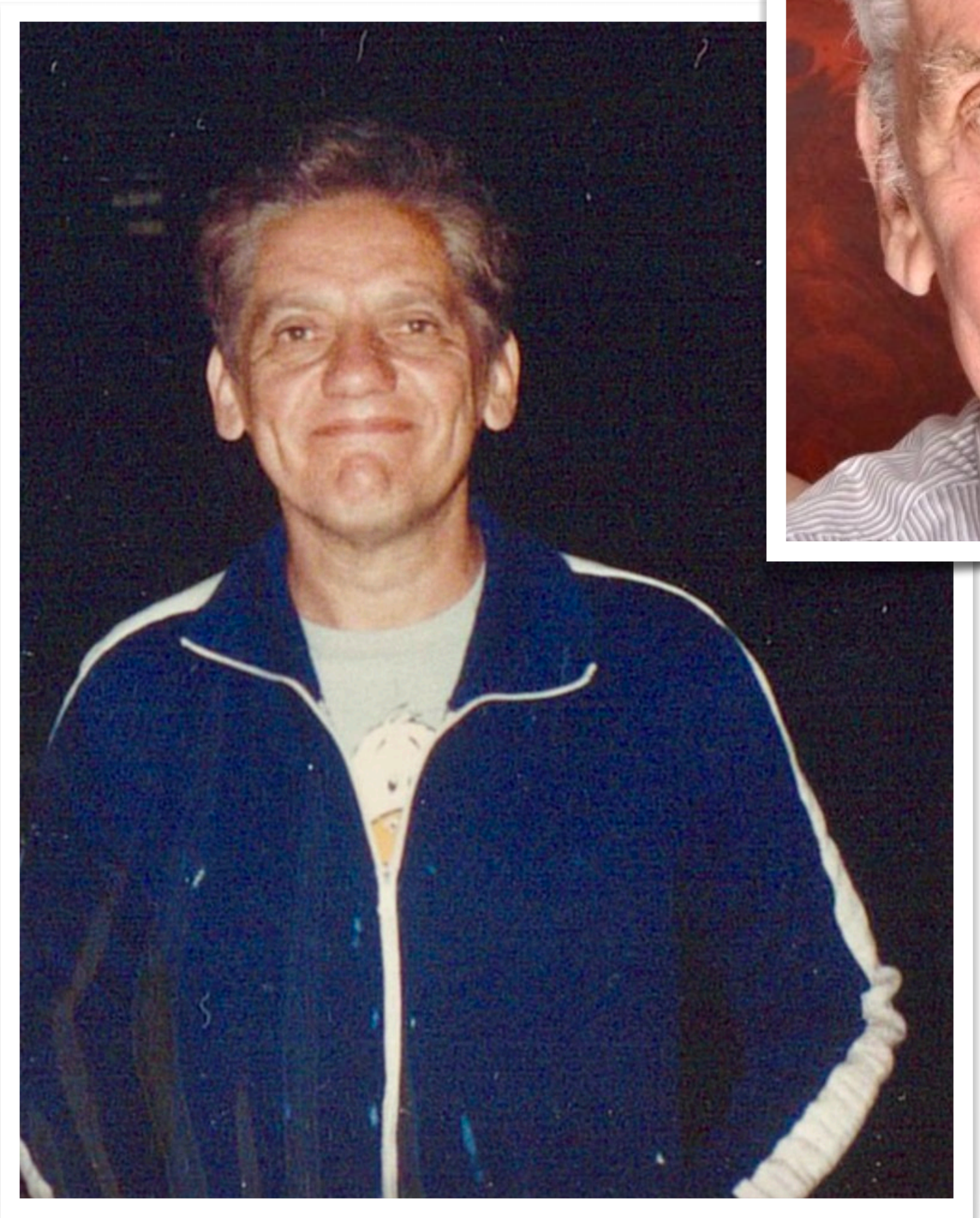
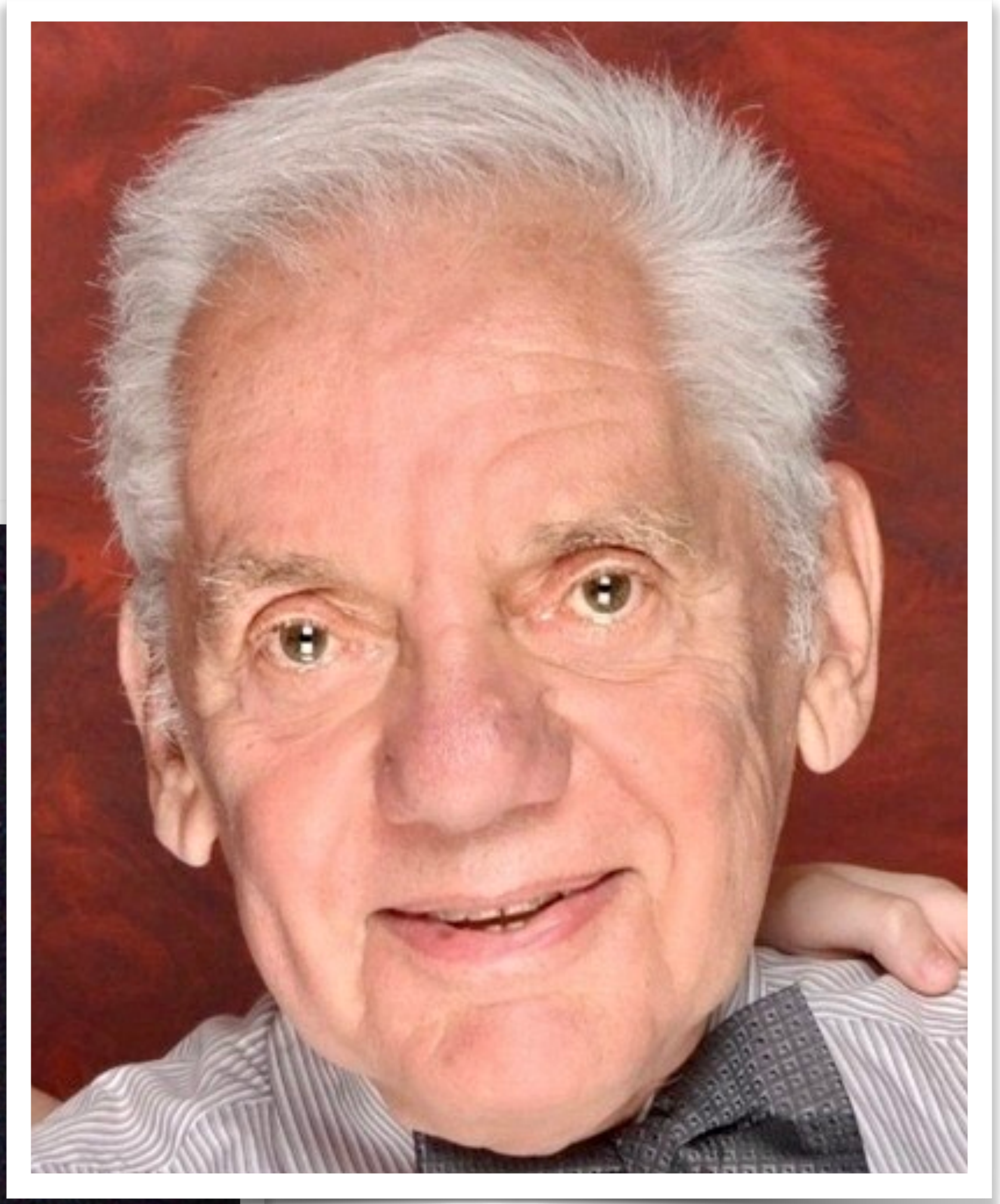
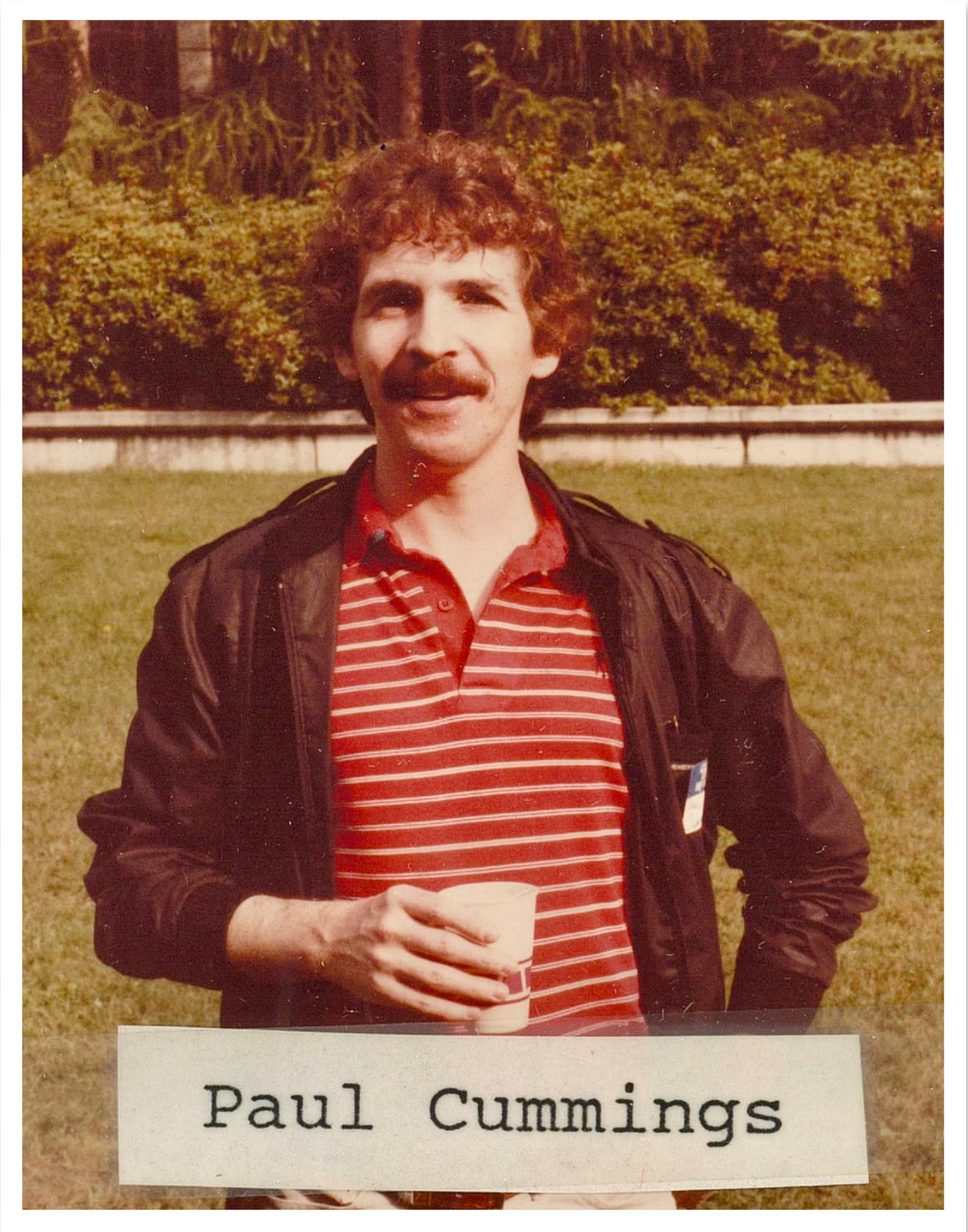
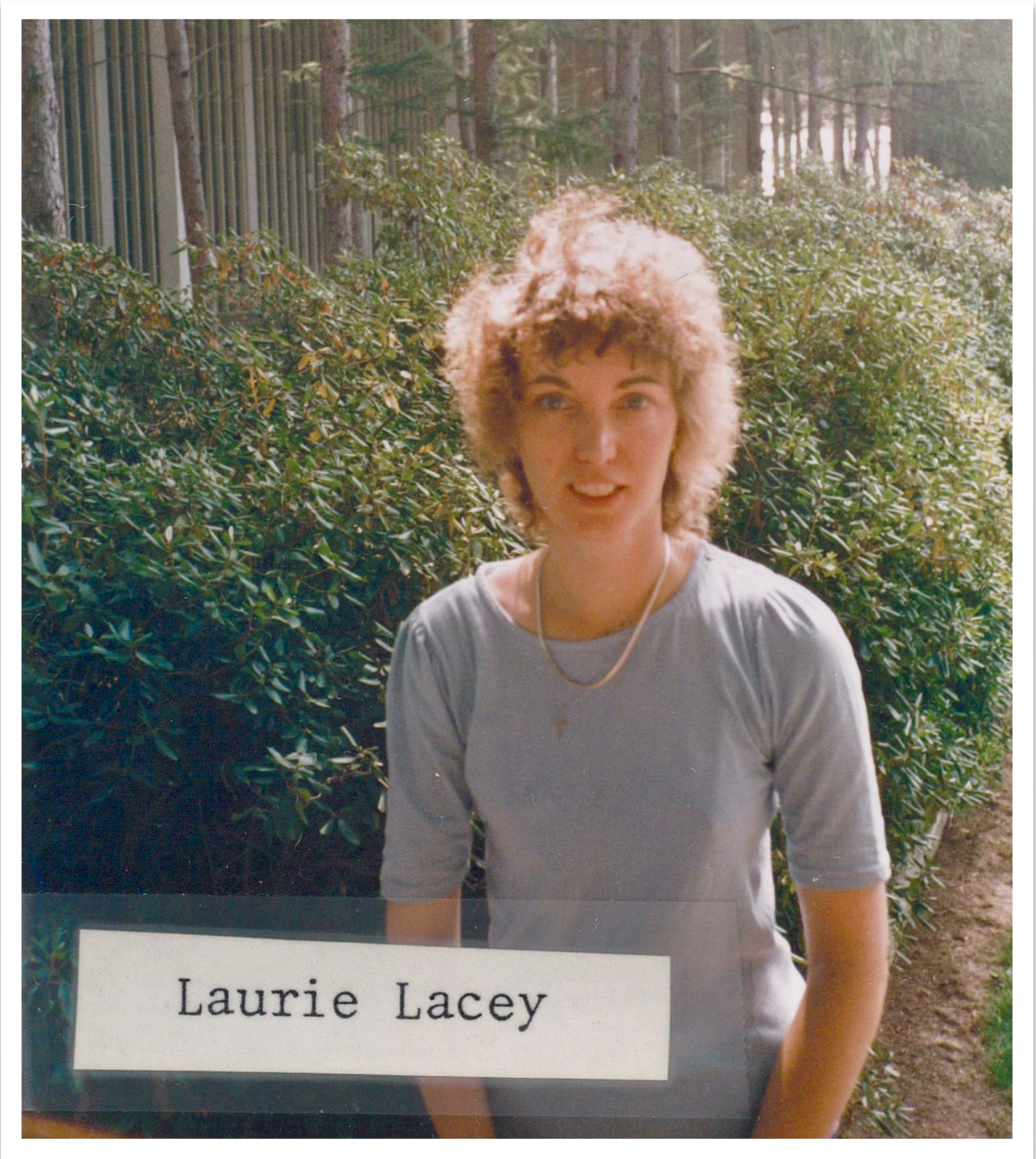

Richard Goldstein



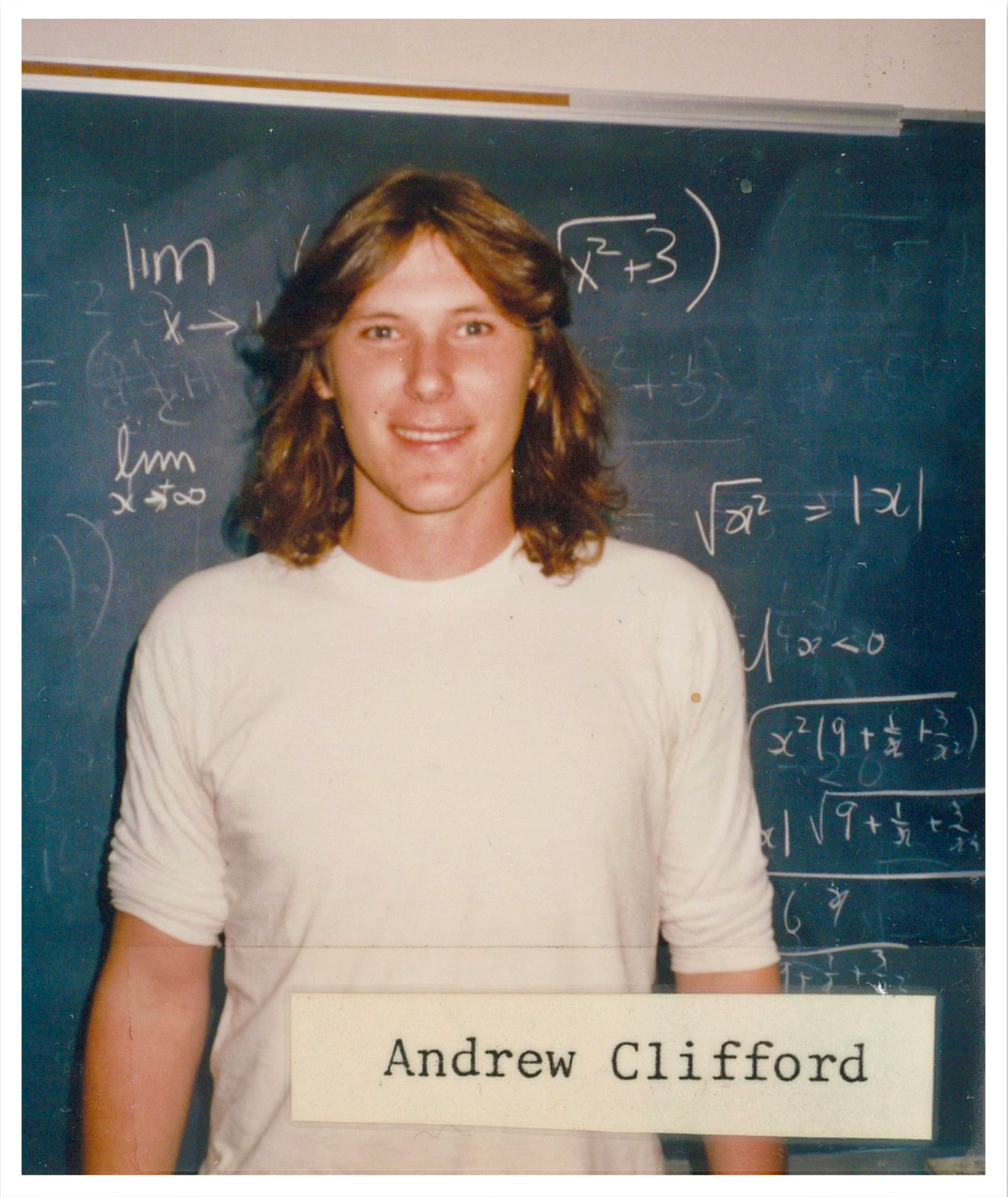




Paul Cummings



Laurie Lacey



Andrew Clifford

Invent. math. 78, 1–12 (1984)

Inventiones mathematicae

Automorphisms of free groups and their fixed points

Richard Z. Goldstein and Edward C. Turner

Department of Mathematics, State University of New York, Albany, NY 12223, USA

§0. Introduction

The use of topological methods in the study of combinational group theory has been extremely fruitful. The identification of free groups as the fundamental groups of graphs has been a particularly useful way of proving theorems about free groups and their subgroups. This point of view was taken by Gersten [2] in his recent proof that the subgroup of words fixed by an automorphism φ of a free group – denoted by Fix (φ) – is finitely generated. In two classic papers, J.H.C. Whitehead described another way of looking at a free group – as the fundamental group of $M_p = \#_1^p S^1 \times S^2$ – that he used to discover striking algorithms to decide:

i) whether a given set of words is part of a basis (in [6]), and
ii) whether two given lists of words are equivalent by an automorphism (in [7]).

UNIVERSITY AT ALBANY
State University of New York

Department of Mathematics and Statistics

Algebra/Topology Seminar

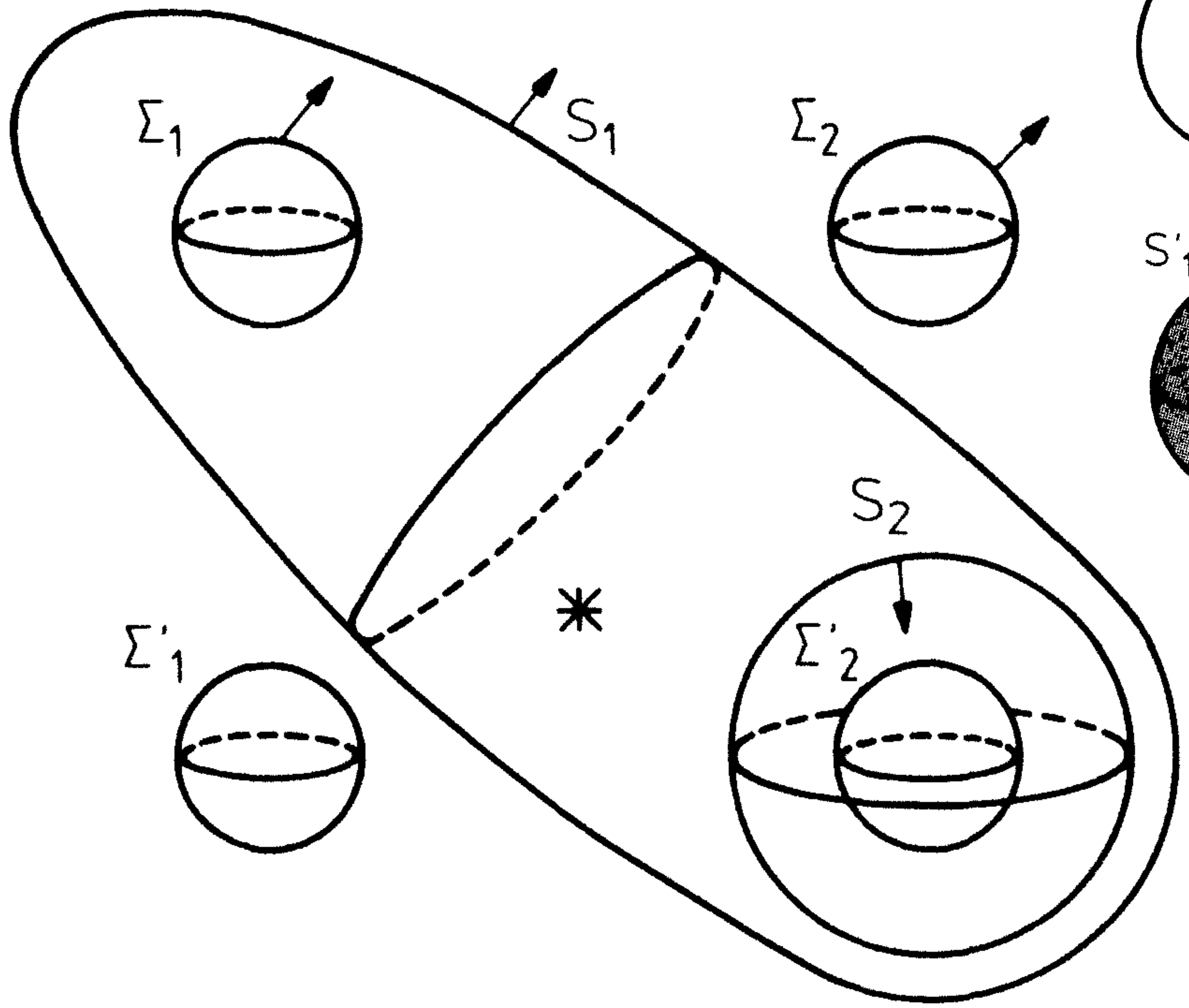
Richard Goldstein

Primitivity in Free Groups

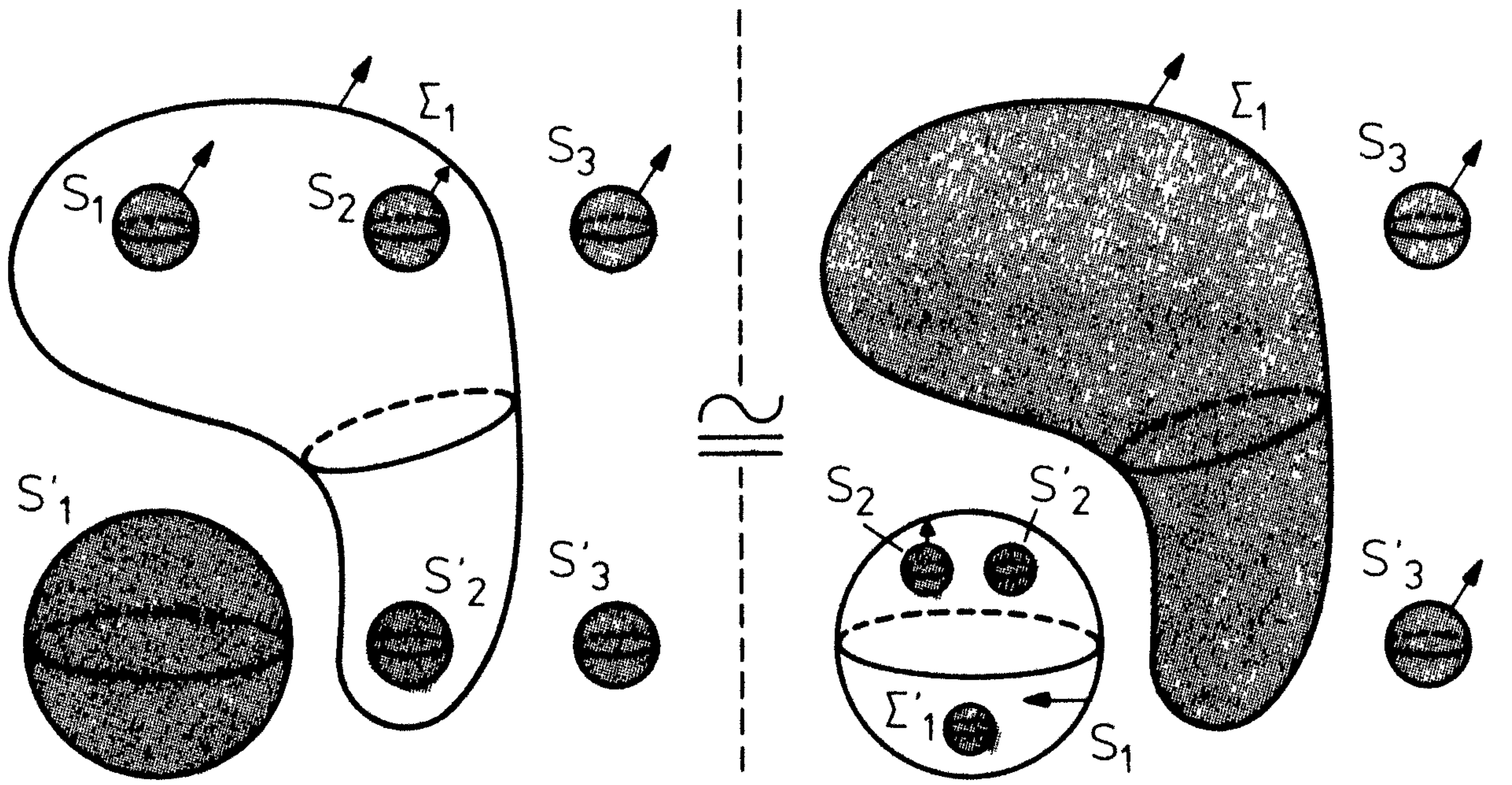
Part 1: Thursday, March 22, 2012, 1:15 p.m. in ES-143

Part 2: Thursday, March 29, 2012, 1:15 p.m. in ES-143

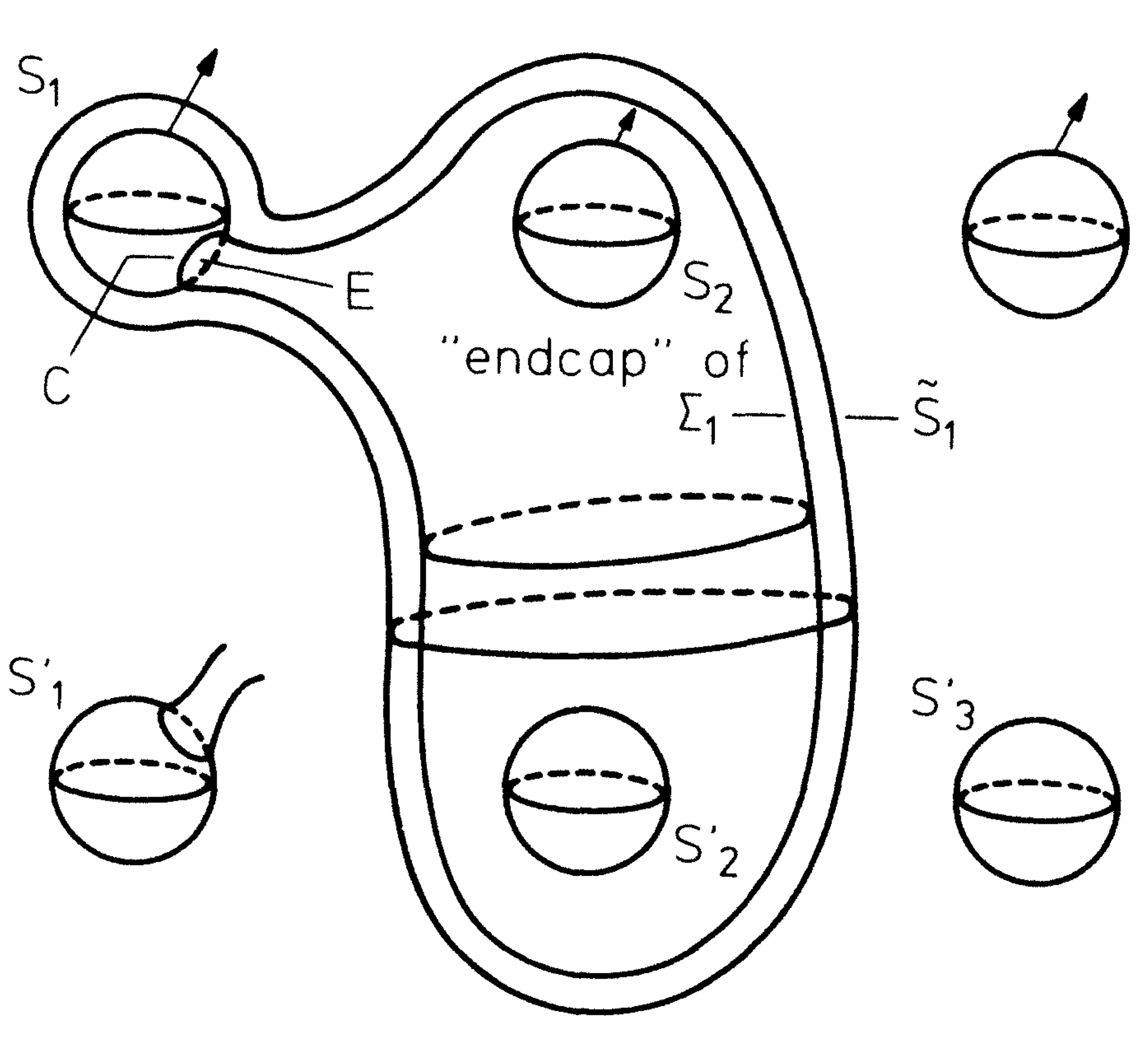
Abstract. A set of elements in a free group F is said to be a primitive set if it is a subset of some basis of F. In these talks several theorems about primitive sets are presented. The results are applications of Whitehead's 3-dimensional model for studying automorphism of free groups; Nielsen transformations; and the folding method of labeled graphs initiated by J. Stallings.



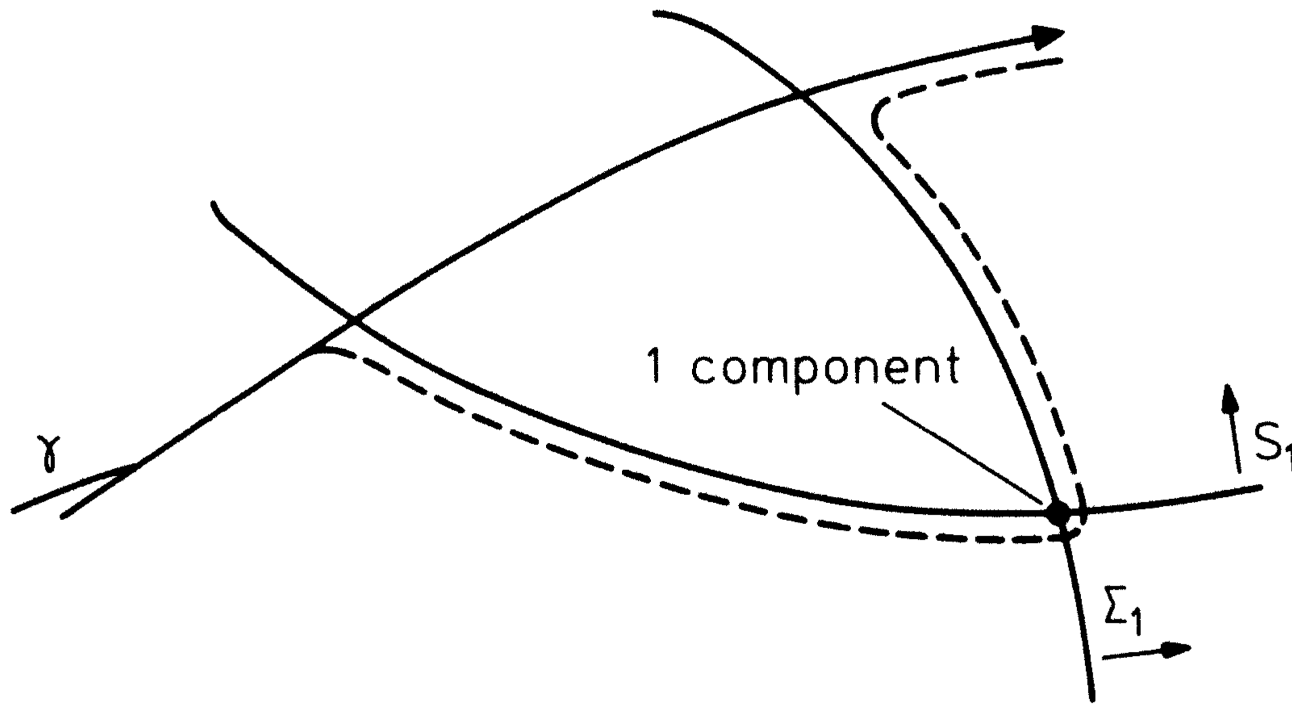
Σ_1
S_1
Σ_2
Σ'_1
S_2
Σ'_2



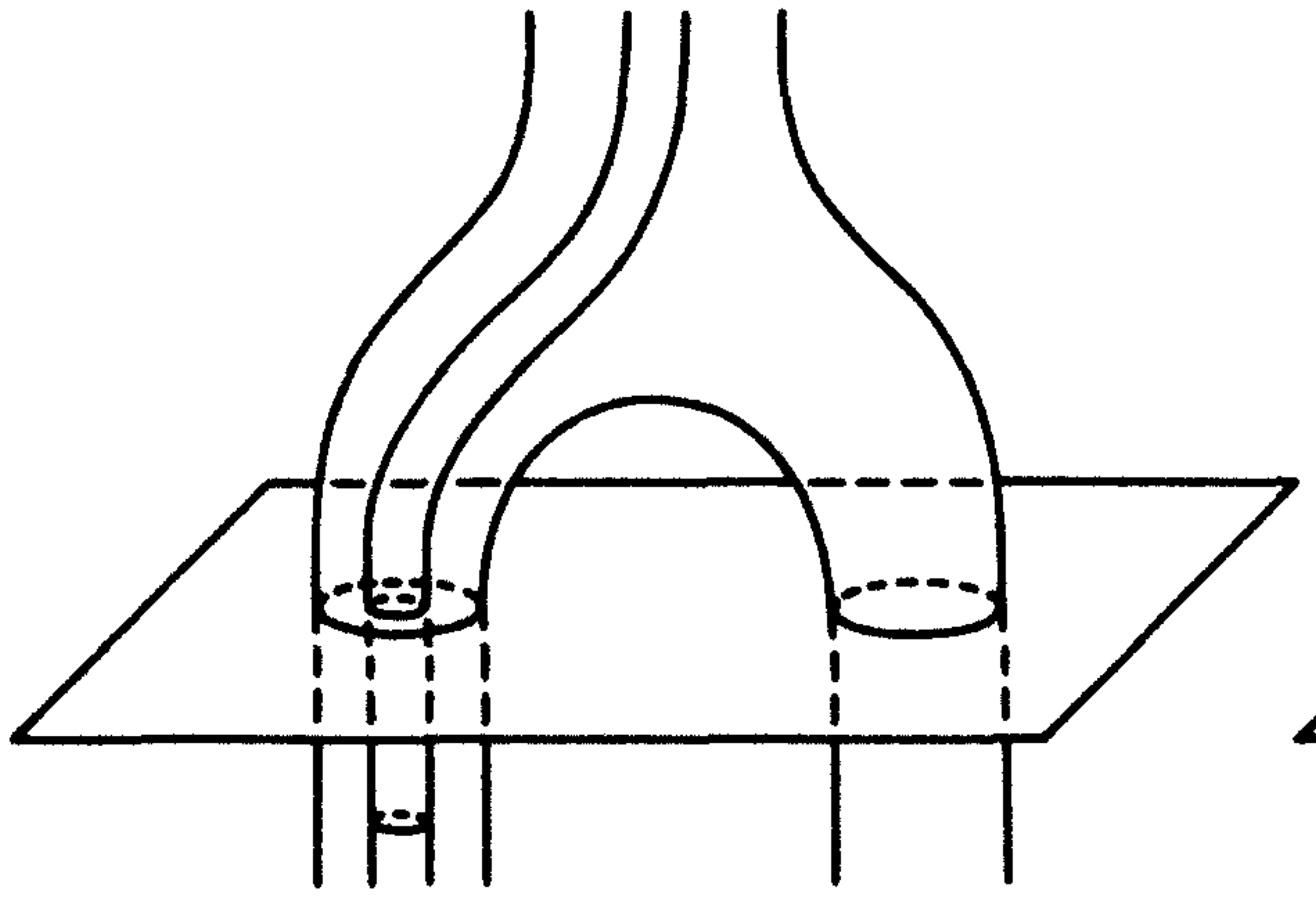
Σ_1
S_1
S_2
S_3
S'_1
S'_2
S'_3
$\cong$
Σ_1
S_3
S_2
S'_2
Σ'_1
S_1
S'_3

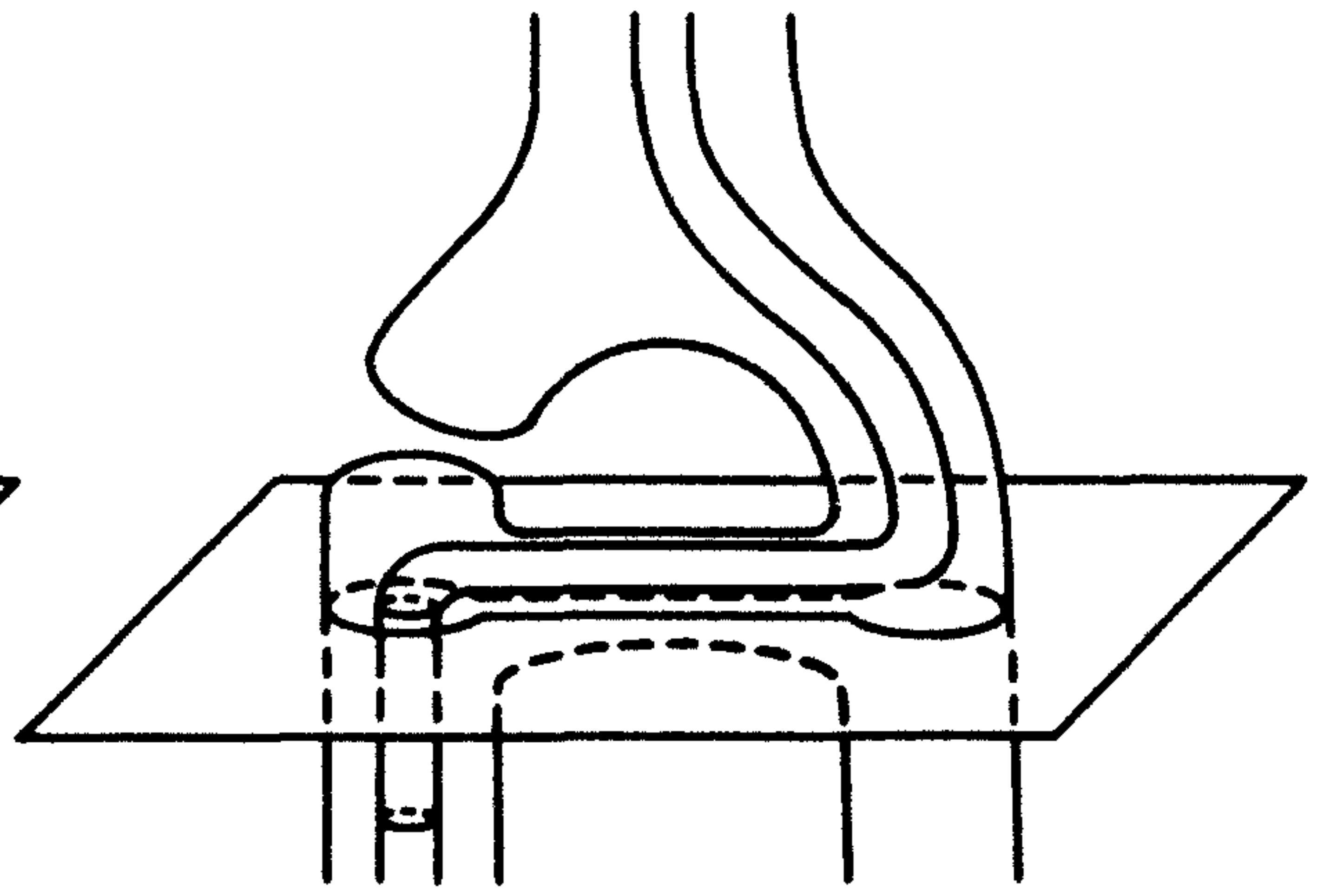


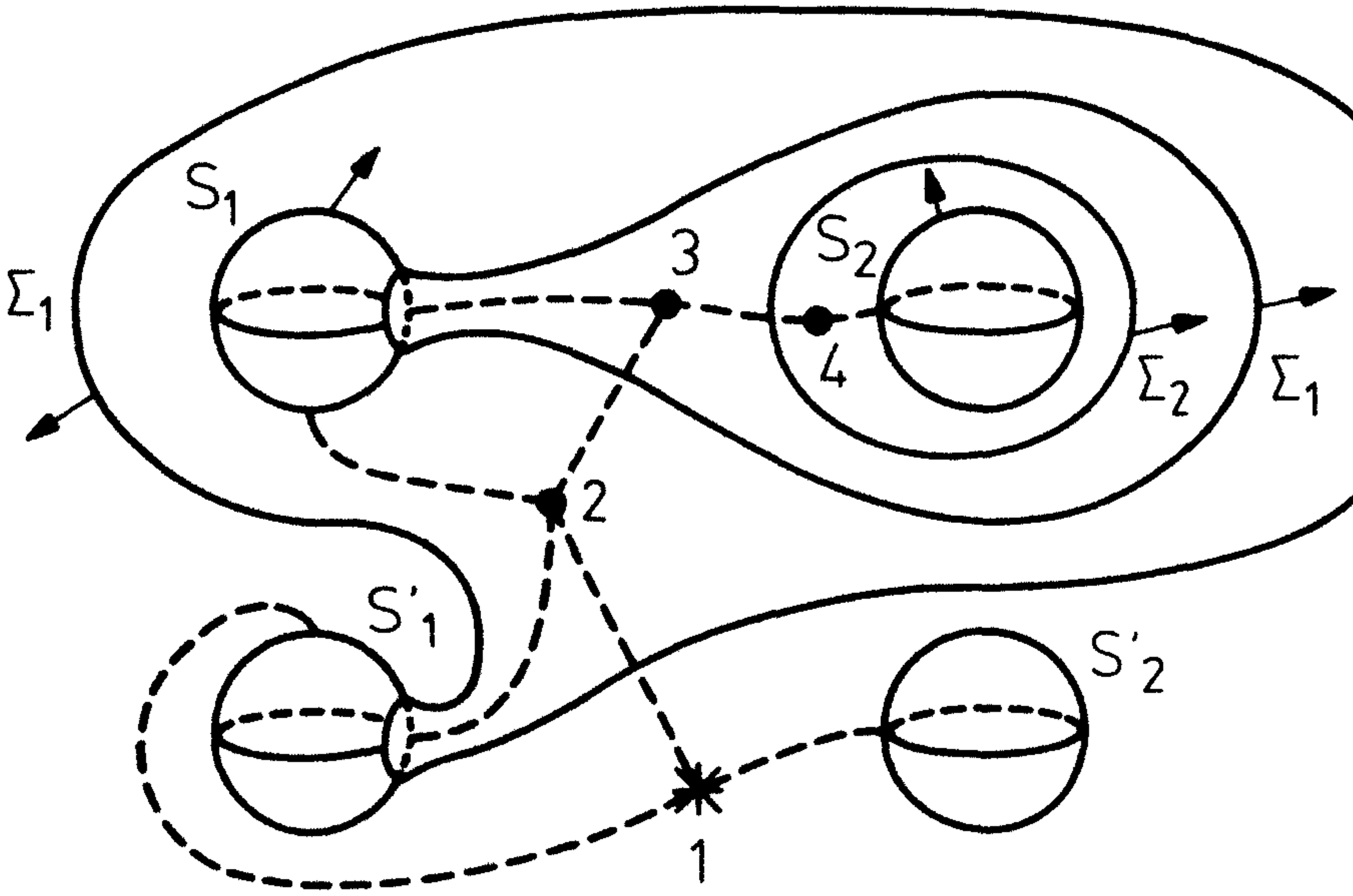
S_1
E
C
S_2
"endcap" of
Σ_1
$\tilde{S}_1$
S'_1
S'_2
S'_3



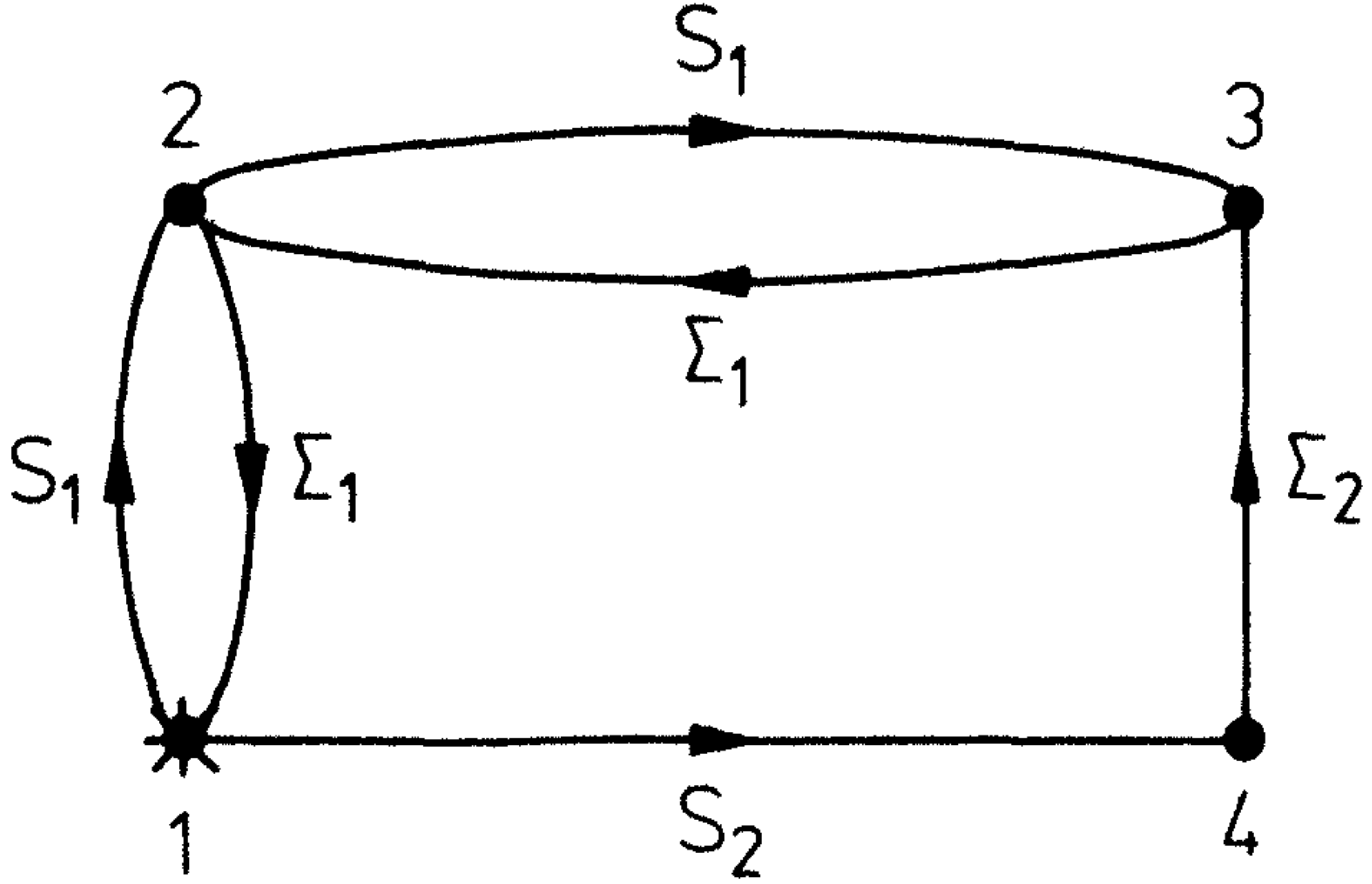
1 component
γ
S_1
Σ_1







S_1
Σ_1
3
S_2
4
Σ_2
Σ_1
2
S'_1
S'_2
1



S_1
2
3
Σ_1
S_1
Σ_1
Σ_2
1
S_2
4

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"The authors' excellent use of drawings [...] makes each definition clear and each step easy to follow, rendering great coherence and clarity to the paper."

Joel M. Cohen

MR0762352 (86h:20031) 20E05 05C25 20E36 57M05
Goldstein, Richard Z. (1-SUNYA); **Turner, Edward C.** (1-SUNYA)
Automorphisms of free groups and their fixed points.
Invent. Math. **78** (1984), *no.* 1, 1–12.

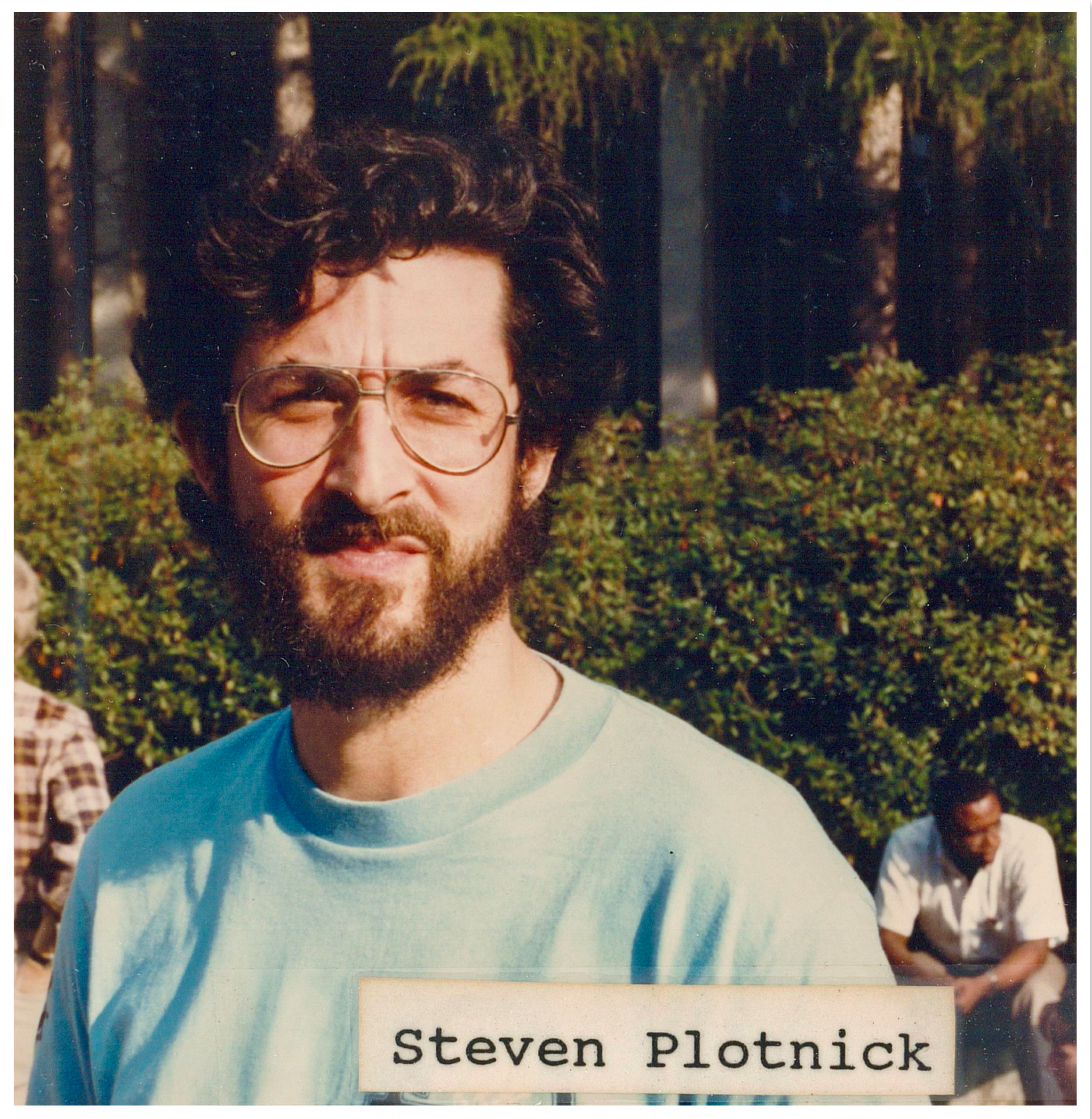
S. Gersten["Fixed points of automorphisms of free groups", Preprint; per bibl.] proved that the subgroup of a finitely generated free group fixed by an automorphism is finitely generated. His point of view was the identification of a free group as the fundamental group of a graph. In the present paper, the authors look at the same problem from another point of view and get some stronger results—a better bound on the number of generators as well as the answer to a problem posed by Gersten [op. cit., Question 9.10].

The point of view here is that taken by J. H. C. Whitehead[Proc. London Math. Soc. **41** (1936), 48–56; Jbuch **62**, 79; Ann. of Math. **37** (1936), 782–800; Zbl **15**, 246] to prove some classical decision algorithms in combinatorial group theory—the identification of a free group with the fundamental group of a connected sum of copies of $S^1 \times S^2$.

More precisely, consider $\{D_i, D_i'\}_{i=1,\cdots,p}$, a set of $2p$ 3-disks in S^3, arbitrary but fixed. Removing the interiors of the disks and identifying the boundaries in pairs gives the space M_p whose fundamental group is F_p, free on p generators $x_1, \cdots, x_p$. M_p now becomes the main tool for carrying out the procedures. Let S be the union of these identified boundaries. If f is an automorphism of F_p, then there exists a corresponding homeomorphism h of M_p. A main step is to show that it is sufficient to assume that S and $\Sigma = h(S)$ sit nicely with respect to one another (the condition of normality).

The precise new results require some technical definitions to describe. The authors' excellent use of drawings, however, makes each definition clear and each step easy to follow, rendering great coherence and clarity to the paper.

Joel M. Cohen


Steven Plotnick

UNIVERSITY AT ALBANY
State University of New York

College of Arts and Sciences
Department of Mathematics and Statistics

Algebra/Topology Seminar

Steve Plotnick

In Memory of Richard Goldstein

Thursday, April 10, 2025
3:00 p.m. in SS-256

Abstract. Richard Goldstein (March 18, 1939 – April 1, 2025) was a professor in our department from 1970 until his retirement in 2018. Among his contributions to geometric topology and combinatorial group theory, the most influential ones were arguably in a series of articles published in the mid 1980's in *Inventiones Mathematicae* and in the *Bulletin of the London Mathematical Society* (authored jointly with Ted Turner, who also was a professor in our department). These articles resolve conjectures of Peter Scott and John Stallings, about finite generation of certain subgroups of free groups, using 3-dimensional manifold topology. This talk will be a friendly and mostly proof-free introduction to these ideas and results.