

Algebra/Topology Seminar

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BETTI CATEGORIES OF GRADED MODULES AND APPLICATIONS TO TORIC IDEALS

Thursday, March 10, 2016

1:15 p.m. in ES-143

ABSTRACT. Graded modules over polynomial rings $\mathbb{k}[x_1, \dots, x_n]$, such as monomial ideals, toric ideals, or defining ideals of projective varieties, for example, can be conveniently thought of as functors from certain small categories to the category of \mathbb{k} -modules. This point of view is also important for the theory of persistent modules in topological data analysis.

In recent joint work with Alex Tchernev, given an arbitrary small category \mathcal{C} , we study functors M from \mathcal{C} to \mathbb{k} -modules, and we describe conditions that guarantee the existence of minimal projective resolutions of such an M . When a minimal projective resolution of M exists, we define a full subcategory of \mathcal{C} , called the Betti category of M , and we prove that the Betti category essentially determines the minimal projective resolution.

This generalizes our previous work for monomial ideals. But, more importantly, we show that these new results can also be applied to toric ideals. The Betti categories thus provide new combinatorial invariants that completely encode the minimal multigraded free resolutions of toric ideals.