Algebra/Topology Seminar

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EXACTNESS AND PROJECTIVITY

Thursday, April 30, 2015
1:15 p.m. in ES-143

ABSTRACT. We consider generalizations to monoidal categories of the tensor-hom relationship for modules over a commutative ring R, i.e., $- \otimes_R M$ is left adjoint to $\mathrm{Hom}_R(M, -)$ as endofunctors of R-Mod. For an object V of a symmetric monoidal category $(\mathcal{V}, \otimes, I, \ldots)$, one can ask whether the endofunctor $- \otimes V$ has a right adjoint. If this is the case, for all V, then $\mathcal{V}$ is called a symmetric monoidal *closed* category. There are many examples of symmetric monoidal closed categories. The existence of left adjoints to $- \otimes V$ is less common in many familiar monoidal categories. We say V is *exact* when such an adjoint exists. For example, in the case of R-Mod, one can show that M is exact if and only if M is finitely generated and projective. The exact commutative R-algebras are precisely those which are exact when considered as R-modules. Our interest in the latter goes back to the late 1970s when we were considering the dual problem for the category of affine schemes over $\mathrm{Spec}R$.

In this talk, we present a generalization of the above characterization of exact R-algebras which applies to several other categories.