



# Algebra/Topology Seminar

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## WORDS WITHIN WORDS A RECURSIVE APPROACH TO THE CATALAN NUMBERS AND FINE NUMBERS

Thursday, April 10, 2014  
1:15 p.m. in ES-143

ABSTRACT. In this talk, I present a new recursive formula for the Catalan numbers. The proof uses Dyck Words, one of the many objects that are known to be counted by the Catalan numbers. It is well-known that the number of Dyck words of length  $2n$  is given by  $C_n$ , the  $n$ -th Catalan number. We obtain our recursion by “factoring” Dyck words into smaller Dyck words which appear inside them. We also present a second proof using generating functions. The generating function approach can be modified to yield a new formula that expresses the  $n$ -th Fine number in terms of the Catalan numbers.