University at Albany, State University of New York

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Algebra/Topology Seminar

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Intersection Pairings on Conley Indices

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Abstract. A vector field on a manifold M generates a flow in M and an invariant set S of the flow is isolated if it is the largest invariant set in some compact neighborhood of itself. Associated to S via the flow is a small category $\mathcal{C}(S)$, the Conley index of S, whose objects are compact pointed spaces called index spaces, and for any two index spaces X and Y, $\mathtt{morph}_{\mathcal{C}(S)}(X,Y)$ consists of a single homotopy class of a flow-defined homotopy equivalence from X to Y. This makes it possible to define reduced singular homology and cohomology modules of a Conley index. Reversing time in the flow (i.e., the vectorfield is replaced by its negative) leaves S an isolated invariant set of the time-reversed flow and associates to S a time-reversed Conly index $\mathcal{C}^*(S)$. In his book *Lectures on Algebraic Topology*, Dold defines intersection numbers and classes for two relative homology classes. I will review this material and use it to define an intersection number pairing on $\left[\tilde{H}_*\mathcal{C}(S)\otimes\tilde{H}_*\mathcal{C}^*(S)\right]_m$ where $m=\dim(M)\geq 2$ and a more general intersection pairing on $\tilde{H}_*\mathcal{C}(S)\otimes\tilde{H}_*\mathcal{C}^*(S)$ with values in the Čech homology of S. I'll outline how the intersection number pairing can be used to prove existence of solution to a two-point boundary value problem arising as the equilibrium equation of the heterozygote inferior case of a Fisher equation, namely,

$$v_t=\varepsilon^2 v_{xx}+v(1-v)(v-a(x)) \quad \text{where } 0<a(x)<1$$

for a single allele in a 1-dimensional habitat with small genetic diffusion within the habitat, i.e., $0<\varepsilon\ll 1$, and no influx of genetic material from outside the habitat, i.e., Neumann boundary conditions. Key to the application is that the intersection pairing is invariant under continuation along a parameterized arc of isolated invariant sets arising from a corresponding arc of vector fields.