UNIVERSITY AT ALBANY
State University of New York

College of Arts and Sciences
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Algebra/Topology Seminar

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Algebraic Manifolds with Vanishing Hodge Cohomology

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1:15 p.m. in ES-143

Abstract. If Y is a complex manifold with $H^i(Y, \Omega_Y^j) = 0$ for all $j \geq 0$ and $i > 0$, then what is Y? Is Y Stein? This is a question raised by J.-P. Serre in 1953. Here Ω_Y^j is the sheaf of holomorphic j-forms and the cohomology is Čech cohomology. Serre's question is open even for a surface. If Y is an algebraic manifold (i.e., an irreducible nonsingular algebraic variety defined over $\mathbb{C}$) of dimension $d \geq 1$ with $H^i(Y, \Omega_Y^j) = 0$ for all $j \geq 0$, $i > 0$, where Ω_Y^j is the sheaf of regular j-forms on Y and $\Omega_Y^0 = \mathcal{O}_Y$, then by Serre duality, we immediately see that Y is not complete. Let X be a smooth completion of Y such that the boundary $X - Y$ is the support of an effective divisor D on X. We may assume that D is a divisor with simple normal crossings by blowing up the closed subset on the boundary $X - Y$ if it is necessary. We show that the Iitaka D-dimension $\kappa(D, X) \neq d - 1$, where $\kappa(D, X)$ is the number of algebraically independent nonconstant rational functions on X with poles in $X - Y$. If d is even, then $\kappa(D, X)$ can be any even number between 0 and d. If d is odd, then $\kappa(D, X)$ can be any odd number between 1 and d. Moreover, if $\kappa(D, X) = d - 2$, then the Kodaira dimension of X is $-\infty$ and if $d > 2$, then $q = h^1(X, \mathcal{O}_X)$ can be any positive integer.