Applied Topology in Albany (ATiA) Seminar

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Order-k Delaunay Mosaics and Alpha Shapes

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ABSTRACT. Given a finite point set in a d-dimensional Euclidean space, order-k Delaunay mosaics, order-k Voronoi tessellations and order-k α-shapes are generalizations of Delaunay triangulations, Voronoi tessellations and α-shapes, respectively. We introduce a $(d+1)$-dimensional geometric cell complex, the rhomboid tiling, which is dual to a well-known hyperplane arrangement, and present properties and relationships between each of the aforementioned notions. Insights about the rhomboid tiling give us a simple and efficient algorithm to compute order-k Delaunay mosaics, and by extension order-k α-shapes which are subcomplexes of the order-k Delaunay mosaic. α-shapes were originally introduced as cell complexes capturing the "shape" of a finite point set and later to compute persistent homology. Their order-k generalizations are more robust against noise, however, this comes at a cost of higher complexity. We explore this increased complexity experimentally using our open-source implementation of the algorithm for order-k Delaunay mosaics. This is joint work with Herbert Edelsbrunner.